Journal of Management and Planning Research
2025, 2(1), 49-60

OLYMPIC MEDAL FORECASTING USING REGRESSION AND TIME-SERIES MODELS

Robin M. Hogarth
Wei Miao
David T.
A. W. Henry

To scientifically estimate medal allocations at the 2028 Los Angeles Olympic Games, this study develops a set of quantitative models, including multiple linear regression, ARIMA time-series analysis, and logistic regression. These models are applied to examine national medal totals, longitudinal medal trends, probabilities of winning a first Olympic medal, and the influence of event composition on medal performance. The analysis is based on historical Olympic data spanning 1984 to 2024. The findings reveal strong linear associations between medal outcomes and key explanatory factors, consistent and reliable forecasting performance from the ARIMA model, and a maximum first-medal probability of 62.5% for Azerbaijan. Additionally, the results demonstrate a positive relationship between the number of contested events and overall medal counts, providing empirical support for strategic planning and resource allocation by National Olympic Committees.

Index Terms — multiple linear regression, arima model, olympic medal prediction

INTRODUCTION

The Olympic Games constitute the apex of international sporting competition, transcending athletic performance to embody geopolitical standing, cultural influence, and national capability [1]. Beyond their symbolic significance, the Games provide a critical context for strategic decision-making in international sports governance, particularly in areas such as resource allocation, athlete development, and long-term competitive planning. The rapid advancement of computational techniques—most notably the maturation of machine learning methods and data science frameworks—has fundamentally reshaped Olympic research. These developments have stimulated growing academic and institutional interest in applying predictive modeling to forecast medal outcomes with increasing analytical precision [2]. As the 2028 Summer Olympic Games in Los Angeles draw nearer, the operational pressures faced by National Olympic Committees (NOCs) intensify. In this context, NOCs confront a multifaceted set of challenges: producing statistically reliable projections of national medal performance, identifying emerging nations with potential for competitive breakthroughs, and assessing the broader systemic effects of changes to the Olympic event programme [3]. Such integrative insights form the empirical foundation of evidence-based policymaking, directly shaping strategic planning, budget allocation, talent identification mechanisms, and the optimal deployment of technical resources across Olympic cycles [4].

Analysis of historical Olympic performance data reveals enduring structural hierarchies within the global medal system. Established sporting powers, most notably the United States and China, continue to demonstrate statistically significant dominance, underpinned by institutionalized training systems, substantial financial investment, and extensive athlete development pipelines. In contrast, the evolving global sporting landscape is characterized by the gradual ascent of developing nations such as Kosovo and Azerbaijan, whose progress reflects targeted specialization in selected sports and strategically focused resource allocation. This interplay between continuity and transformation in Olympic competition is shaped by a complex constellation of factors, including historical performance trends, sport-specific technological advancements, national investment strategies, host-nation advantages, and, critically, the configuration of the Olympic sports programme itself. The addition or removal of events—driven by considerations such as technological relevance, media appeal, or gender equity—can generate cascading effects across national medal prospects. Accordingly, the construction of advanced computational frameworks capable of integrating these interdependent variables is methodologically essential. Only through systematic, model-based analysis can stakeholders obtain the analytical depth required to navigate the increasingly dynamic landscape of global sporting competition and convert predictive insights into actionable strategic advantage.

Existing research on Olympic medal prediction reflects a distinctly multidisciplinary orientation. Schlembach [5] developed a machine learning framework grounded in socio-economic indicators using a two-stage random forest algorithm, demonstrating strong predictive accuracy for total, non-zero, and zero medal counts across multiple evaluation metrics. Makiyan [6] advanced medal forecasting models from an economic perspective, highlighting the value of interdisciplinary integration in predictive analyses. In parallel, Forrest [7] and Zhao [8] conducted comparative evaluations of forecasting approaches from statistical and econometric viewpoints. Nevertheless, much of the existing literature remains centered on single-model approaches, while local contextual factors and comprehensive analyses of medal distributions and award dynamics remain insufficiently explored.

In response to these limitations, the present study develops a multi-model forecasting framework based on multiple linear regression and ARIMA time-series analysis to address the following objectives. First, the study forecasts the medal table for the 2028 Summer Olympic Games in Los Angeles by analyzing historical medal data and modeling country-level medal trajectories using ARIMA techniques. Second, it estimates the probability of countries winning Olympic medals for the first time. Third, it examines the underlying factors contributing to the initial emergence of new medal-winning nations. Finally, the results across models are integrated to identify systematic patterns in medal distribution and to formulate strategic recommendations for National Olympic Committees.

MATERIALS AND METHODS

Data Acquisition and Preprocessing

Data acquisition

The scope of this study covers Olympic medal outcomes, athlete participation levels, event programme configurations, and host-country information for each participating nation from 1984 to 2024. The dataset was compiled from two principal sources: the Olympic Movement's central database and publicly available reports released by National Olympic Committees. These sources jointly provide comprehensive and authoritative records suitable for longitudinal and cross-national analysis.

Data preprocessing

To ensure data reliability, modeling stability, and analytical robustness, a systematic preprocessing procedure was implemented. Potential outliers were first identified using the interquartile range (IQR) method [9], which effectively mitigates the influence of extreme observations, such as anomalies arising from newly introduced or discontinued Olympic events. This procedure prevents such irregularities from disproportionately distorting overall statistical patterns.

Following outlier detection, identified extreme values were replaced with the median of the corresponding sport category. Median substitution has been shown to reduce the impact of outliers while preserving the underlying distributional structure and stability of the dataset. A secondary validation using the quartile distance method was subsequently conducted, and the resulting box-and-whisker plots confirmed that all processed observations lay within acceptable bounds [10]. The outcomes of the preprocessing procedure are illustrated in Figures 1.

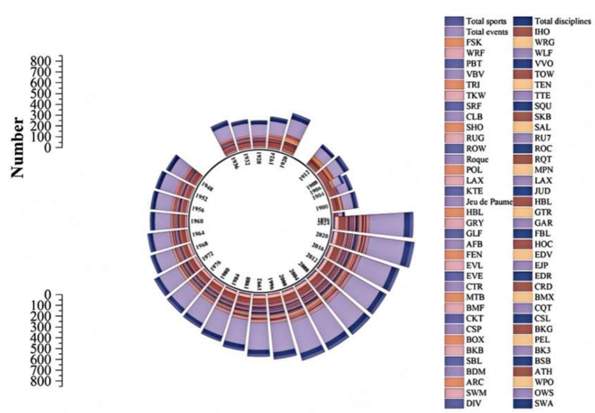

Figure 1: Spiral bar chart after missing values are filled

Correlation analysis of characteristic variables

Given the large number of explanatory variables potentially affecting medal outcomes, it is essential to examine inter-variable relationships prior to model construction. This step ensures the selection of appropriate modeling strategies and enhances the interpretability of subsequent results. The characteristic variables considered include Rank, Gold, Silver, Bronze, Total medals, and Year.

Before conducting correlation analysis, a normality assessment was performed using quantile–quantile (Q–Q) plots. The results indicated substantial deviations between observed and theoretical quantiles for all variables, confirming the presence of non-normal distributions. Consequently, Spearman's rank correlation coefficient was selected as the most suitable non-parametric measure of association [11].

Spearman correlation analysis requires ranking each variable and computing the differences between paired ranks. The coefficient is defined as:

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}, \tag{1}$$

where r_s denotes the Spearman rank correlation coefficient, d_i represents the difference between the ranks of paired observations, and n is the sample size.

The resulting correlation heat map, presented in Figure 2, reveals strong positive correlations between gold, silver, and bronze medal counts and the total number of medals, indicating that these components tend to increase synchronously. A pronounced negative correlation is observed between rank and total medal count, implying that higher-ranked countries generally accumulate more medals. In contrast, the relationship between the year variable and medal counts is relatively weak. These findings identify gold, silver, and bronze medals as primary explanatory variables for subsequent prediction models. To forecast future national medal totals, multiple linear regression is therefore adopted as an effective analytical approach [12].

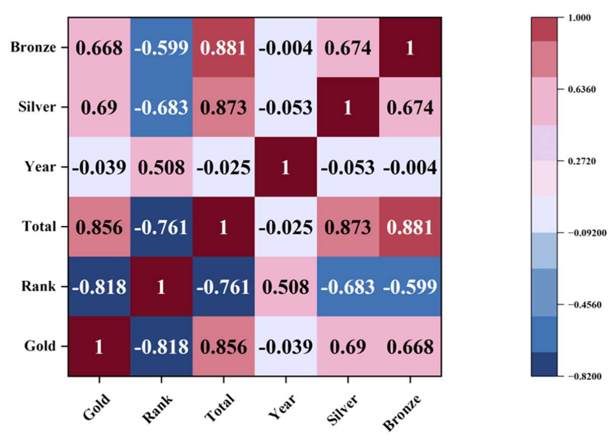

Figure 2: Heat map of correlation analysis results of data features

Methodology

Multiple linear regression model

Multiple linear regression offers several advantages, including conceptual simplicity, interpretability, and computational efficiency. Preliminary data analysis indicates a predominantly linear relationship between medal-related explanatory variables and actual medal outcomes. Accordingly, a multiple linear regression framework was employed to predict both gold medal counts and total medal counts for each country at the 2028 Olympic Games.

The composition of Olympic sports plays a pivotal role in shaping national medal distributions. An increase in the number of events typically leads to higher total medal counts, while the introduction of new disciplines may alter competitive dynamics. For example, established sporting powers may gain early advantages in emerging events, or host-selected disciplines may favor the host nation. By incorporating variables related to the number and type of events, the multiple linear regression model quantifies their effects on gold and total medal counts, thereby offering actionable insights to support strategic planning by National Olympic Committees.

ARIMA model building and solving

To capture the temporal dependencies inherent in historical medal data, the Autoregressive Integrated Moving Average (ARIMA) model was employed. This time-series approach is well suited for identifying and extrapolating long-term trends, cyclical behavior, and stochastic fluctuations present in longitudinal Olympic records. Its suitability for non-stationary data with serial correlation makes it particularly effective for forecasting medal trajectories leading up to the 2028 Games.

The ARIMA framework consists of three core components: autoregression (AR), which models linear dependence on past values; integration (I), which applies differencing to remove trends and stabilize the series; and moving average (MA), which represents the influence of past error terms through weighted averages. Together, these components enable robust modeling of medal count evolution over time.

Logistic regression model building and solving

To estimate the likelihood that a previously non-medal-winning country will secure its first Olympic medal at the 2028 Los Angeles Games, a logistic regression classifier was constructed. This binary classification model defines the dependent variable as whether a country wins at least one medal, enabling probabilistic predictions of first-time medal success.

Model parameters are estimated by maximizing the likelihood function, ensuring that predicted probabilities closely align with observed outcomes. Through iterative optimization, the logistic regression model adjusts its internal coefficients until convergence is achieved. In this context, the likelihood function represents the probability of observing the given medal outcomes conditional on the explanatory variables.

Model Evaluation Metrics

The number and composition of Olympic sports have been shown to significantly influence both medal distribution patterns and aggregate medal totals. While an increase in events generally expands overall medal counts, newly introduced sports may alter competitive structures and shift medal advantages. These effects can be rigorously quantified using multiple linear regression, enabling National Olympic Committees to formulate evidence-based strategic policies [13].

To evaluate regression model performance, the coefficient of determination (R^2) and the mean squared error (MSE) are employed as complementary metrics. R^2 measures the proportion of variance explained by the model, while MSE captures average prediction error magnitude. These metrics are defined as:

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2}, \tag{2}$$

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2, \tag{3}$$

where y_i denotes observed values, $\hat{y}_i$ predicted values, $\bar{y}$ the sample mean, and N the number of observations.

In this study, the regression model achieves an R^2 value of 0.86 and an MSE of 16.58. Given the magnitude of medal count data, an R^2 close to unity and a relatively small MSE indicate a strong model fit and high predictive accuracy. Although some degree of non-linearity may persist, the multiple linear regression framework demonstrates robust performance in forecasting Olympic medal outcomes.

RESULTS AND ANALYSIS

Least Squares Estimation and Multiple Linear Regression

Model establishment

The multiple linear regression model is formulated as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \cdots + \beta_n X_n + \varepsilon, \tag{4}$$

where Y denotes the predicted number of gold medals or total medals; $X_1, X_2, \ldots, X_n$ represent the explanatory variables, including factors such as historical medal performance, national rankings, athlete participation, and the number of events; β_0 is the intercept term reflecting the baseline medal count when all explanatory variables are zero; $\beta_1, \beta_2, \ldots, \beta_n$ are regression coefficients measuring the magnitude and direction of each variable's influence; and ε is the stochastic error term.

The sign and magnitude of each regression coefficient provide insight into the relative contribution of the corresponding variable to medal outcomes. By fitting the regression model to the training dataset and estimating optimal coefficients, the objective is to minimize the discrepancy between predicted and observed medal counts, thereby achieving an accurate and reliable predictive model [14].

Least squares estimation and regression solution

Ordinary least squares (OLS) estimation is employed to determine the regression parameters. The objective function is defined as

$$J(\beta) = \sum_{i=1}^{N}\left[y_i - (\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_n x_{ni})\right]^2, \tag{5}$$

where y_i is the observed medal count for the ith sample and the expression in parentheses represents the model's predicted value.

Minimizing the sum of squared residuals yields the closed-form solution for the regression coefficients:

$$\beta = (X^T X)^{-1} X^T Y, \tag{6}$$

where $\beta = (\beta_0, \beta_1, \ldots, \beta_n)^T$ is the coefficient vector, X is the $N \times (n+1)$ design matrix with a leading column of ones corresponding to the intercept, Y is the $N \times 1$ vector of observed medal counts, and X^T denotes the transpose of X.

Once the coefficients are estimated, medal counts for new observations X' can be predicted as

$$Y' = X'\beta. \tag{7}$$

Regression results and prediction

Figure 3 presents the predicted medal standings for the 2028 Los Angeles Summer Olympics, displaying the top 20 countries in both total medals and gold medals, together with projection intervals for all participating nations [15]. While the linear regression model demonstrates strong predictive capability, potential nonlinear relationships may persist. Incorporating logarithmic transformations or polynomial extensions in future model refinements may further enhance predictive accuracy.

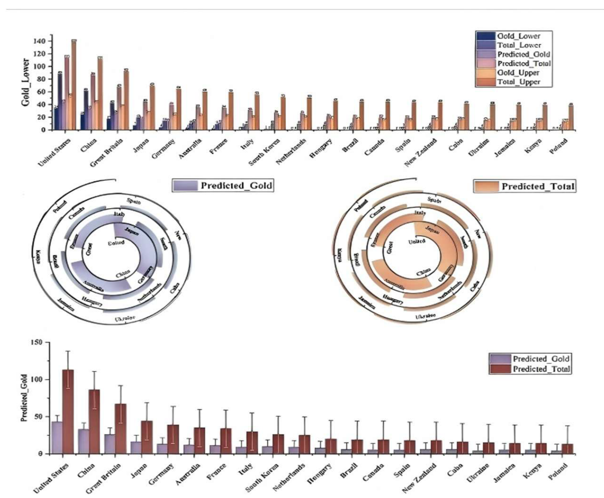

Figure 3: Projected results of the number of medals, gold medals and their projection intervals

ARIMA-Based Time-Series Analysis

Model specification

The ARIMA model is expressed as

$$Y_t = c + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t, \tag{8}$$

where Y_t denotes the observed medal count at time t, p and q are the orders of the autoregressive (AR) and moving average (MA) components, respectively, ϕ_i and θ_j are the corresponding coefficients, ε_t represents white noise, and c is a constant term.

To ensure stationarity, model smoothness is tested using unit root diagnostics. If the p-value exceeds the conventional significance threshold of 0.05, differencing is applied. First-order differencing is defined as

$$Y_t' = Y_t - Y_{t-1}. \tag{9}$$

The augmented Dickey–Fuller (ADF) test is iteratively applied until stationarity is achieved [16].

Visualization of medal trends

Figure 4 compares predicted medal counts for the top 20 countries at the 2028 Olympics with their historical averages. The shaded area represents the deviation between predicted and historical values. Kosovo exhibits the most pronounced positive deviation, indicating substantial potential improvement, while Spain shows the largest projected decline. Such deviations provide insights into possible performance shifts across nations.

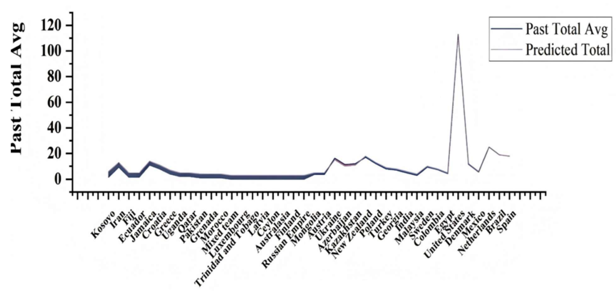

Figure 4: Two-color plot of national medal trends

Logistic Regression Analysis

Model formulation

To estimate the probability that a country will win a medal for the first time in 2028, a logistic regression model is constructed:

$$P(\text{Winning a medal}) = \frac{1}{1+e^{-(\beta_0+\beta_1X_1+\beta_2X_2+\cdots+\beta_nX_n)}}, \tag{10}$$

where $X_1, X_2, \ldots, X_n$ are explanatory variables influencing medal attainment and $\beta_0, \beta_1, \ldots, \beta_n$ are coefficients estimated from the data. The logistic function constrains predicted probabilities to the interval $[0,1]$.

Maximum likelihood estimation

Regression coefficients are obtained by maximizing the likelihood function

$$L(\beta) = \prod_{i=1}^{n} P(y_i \mid X_i), \tag{11}$$

where $y_i \in \{0,1\}$ indicates whether country i has won a medal. The probability term is defined as

$$P(y_i \mid X_i) = [P(\text{Winning a medal})]^{y_i} [1 - P(\text{Winning a medal})]^{1-y_i}. \tag{12}$$

Taking logarithms yields the log-likelihood function

$$\ell(\beta) = \sum_{i=1}^{n} [y_i \log P(\text{Winning a medal}) + (1-y_i)\log(1 - P(\text{Winning a medal}))], \tag{13}$$

which is maximized using gradient descent until convergence [18].

Probability prediction and visualization

For new observations, the probability of medal attainment is computed as

$$P(\text{Winning a medal} \mid X) = \frac{1}{1+e^{-(\beta_0+\beta_1 X_1+\cdots+\beta_n X_n)}}. \tag{14}$$

Figures 5 visualize the ratio of historical medals to Olympic participations and the predicted probabilities of first-time medal success, respectively. Azerbaijan emerges as the leading candidate with a predicted probability of 62.5%, substantially higher than other nations. This result reflects sustained improvements in competitive performance and targeted investment strategies quantified by the model [17].

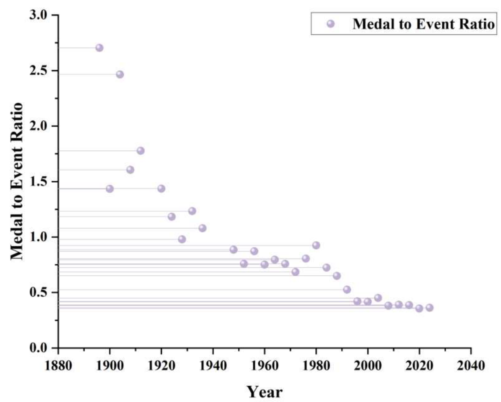

Figure 5: Scatterplot of ratio of number of medals to number of events

Event Structure and Medal Distribution Analysis

Model establishment

To examine the influence of Olympic programme configuration on medal outcomes, a multivariate linear regression model is specified as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n + \varepsilon, \tag{15}$$

where Y represents gold or total medal counts and $X_1, X_2, \ldots, X_n$ denote programme-related variables such as the number, type, and difficulty of events [19].

Estimation and results

Least squares estimation minimizes the sum of squared residuals:

$$\hat{y}_i = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_n x_{ni}, \tag{16}$$

$$\varepsilon_i = y_i - \hat{y}_i, \tag{17}$$

$$F(\beta) = \sum_{i=1}^{N} \varepsilon_i^2. \tag{18}$$

The coefficient estimates are obtained as

$$\hat{\beta} = (X^T X)^{-1} X^T Y. \tag{19}$$

Visualization and interpretation

Figure 6 illustrates a positive relationship between the number of events and gold medal counts, particularly for traditionally strong nations. However, newly introduced disciplines such as sport climbing, skateboarding, and surfing exhibit limited immediate impact on established medal hierarchies [21]. This suggests that programme changes require country-specific evaluation aligned with existing strengths and investment strategies.

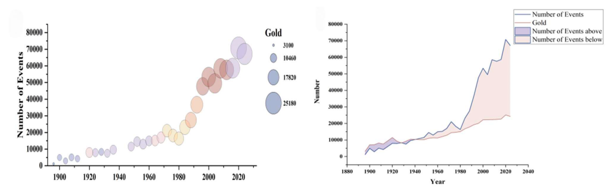

Figure 6: Plot of Number of Gold Medals vs. Number of Events

Accordingly, National Olympic Committees should conduct systematic assessments of programme modifications to identify disciplines offering high marginal medal returns, events misaligned with national capabilities, and emerging sports presenting opportunities for competitive breakthroughs [22]. In parallel, the analysis highlights the pivotal role of elite coaching as a performance multiplier. Multivariate regression results indicate that coaching quality exerts a measurable influence on medal outcomes, operating through technical refinement, psychological preparation, and tactical optimization [23]. Recognizing and quantifying this effect enables evidence-based strategies for coach recruitment, development, and retention, reinforcing coaching excellence as a deliberate driver of Olympic success.

CONCLUSION

This study develops an integrated, data-driven framework for forecasting Olympic medal outcomes, combining multiple linear regression, ARIMA time-series analysis, and logistic regression to examine medal distributions, temporal trends, first-time medal probabilities, and the structural effects of Olympic programme design. Using historical data from 1984 to 2024, the results demonstrate that medal outcomes exhibit strong and stable relationships with key explanatory variables, validating the suitability of regression-based approaches for national-level medal prediction.

The multiple linear regression models reveal a robust positive association between medal counts and factors such as historical performance, athlete participation, and the number of competition events, highlighting the central role of programme scale in shaping medal opportunities. ARIMA-based analysis further captures longitudinal dynamics in national medal trajectories, identifying countries with potential performance improvements or regressions ahead of the 2028 Los Angeles Olympic Games. In parallel, the logistic regression model provides probabilistic insight into first-time medal attainment, with results indicating that countries such as Azerbaijan possess a notably high likelihood of achieving inaugural Olympic success.

Beyond predictive accuracy, the findings offer substantive strategic implications for National Olympic Committees. Programme modifications, including the introduction of new sports, do not uniformly alter established medal hierarchies and therefore require nation-specific evaluation aligned with existing competitive strengths and investment capacities. Moreover, the analysis underscores the measurable contribution of elite coaching and targeted resource allocation as performance multipliers capable of enhancing medal efficiency.

Overall, this study demonstrates that a multi-model analytical approach can effectively translate complex historical Olympic data into actionable strategic intelligence. By integrating predictive modeling with structural and probabilistic analysis, the proposed framework supports evidence-based decision-making in athlete development, resource prioritization, and long-term Olympic planning, thereby providing a practical and adaptable tool for navigating the evolving landscape of global Olympic competition.

REFERENCES

[1] MacAloon J J. Olympic Games and the theory of spectacle in modern societies, The Olympics. Routledge, 2023: 80-107.

[2] Sahu M, Gupta R, Ambasta R K, et al. Artificial intelligence and machine learning in precision medicine: A paradigm shift in big data analysis. Progress in molecular biology and translational science, 2022, 190(1): 57-100.

[3] Mair J, Chien P M, Kelly S J, et al. Social impacts of mega-events: A systematic narrative review and research agenda. Methodological Advancements in Social Impacts of Tourism Research, 2023: 140-162.

[4] Lopes dos Santos G, Gonçalves J. The Olympic Effect in strategic planning: insights from candidate cities. Planning Perspectives, 2022, 37(4): 659-683.

[5] Schlembach C, Schmidt S L, Schreyer D, et al. Forecasting the Olympic medal distribution–a socioeconomic machine learning model. Technological Forecasting and Social Change, 2022, 175: 121314.Johnson, D. (2024). Olympic Medal Prediction Model. Colorado College.

[6] Makiyan S, Rostami M. Economic determinants of success in Olympic Games. Turkish Journal of Sport and Exercise, 2021, 23(1): 33-39.

[7] Forrest D, Tena J D, Varela-Quintana C. The influence of schooling on performance in chess and at the Olympics. Empirical Economics, 2023, 64(2): 959-982.

[8] Zhao S, Cao J, Steve J. Research on Olympic medal prediction based on GA-BP and logistic regression model. F1000Research, 2025, 14: 245.

[9] Alabrah A. An improved CCF detector to handle the problem of class imbalance with outlier normalization using IQR method. Sensors, 2023, 23(9): 4406.

[10] Wang H, Li J, Li Z. AI-generated text detection and classification based on BERT deep learning algorithm. arXiv preprint arXiv:2405.16422, 2024.

[11] Ali Abd Al-Hameed K. Spearman's correlation coefficient in statistical analysis. International Journal of Nonlinear Analysis and Applications, 2022, 13(1): 3249-3255.

[12] Öztürk O B, Başar E. Multiple linear regression analysis and artificial neural networks based decision support system for energy efficiency in shipping. Ocean Engineering, 2022, 243: 110209.

[13] Das A. Logistic regression, Encyclopedia of quality of life and well-being research. Cham: Springer International Publishing, 2024: 3985- 3986.

[14] Valentini M, dos Santos G B, Muller Vieira B. Multiple linear regression analysis (MLR) applied for modeling a new WQI equation for monitoring the water quality of Mirim Lagoon, in the state of Rio Grande do Sul—Brazil. SN Applied Sciences, 2021, 3: 1-11.

[15] Huimin S H I, Dongying Z, Yonghui Z. Can Olympic Medals Be Predicted? Based on the Interpretable Machine Learning Perspective. Journal of Shanghai University of Sport, 2024, 48(4): 26-36.

[16] Gil-Alana L A, Yaya O O S. Testing fractional unit roots with non-linear smooth break approximations using Fourier functions. Journal of Applied Statistics, 2021, 48(13-15): 2542-2559.

[17] Vrugt J A, de Oliveira D Y, Schoups G, et al. On the use of distribution-adaptive likelihood functions: Generalized and universal likelihood functions, scoring rules and multi-criteria ranking. Journal of Hydrology, 2022, 615: 128542.

[18] Haji S H, Abdulazeez A M. Comparison of optimization techniques based on gradient descent algorithm: A review. PalArch's Journal of Archaeology of Egypt/Egyptology, 2021, 18(4): 2715-2743.

[19] Arpna N, Dalal S. Optimizing Linear Regression model. Trends in Mechatronics Systems: Industry 4. 0 Perspectives, 2024: 73.

[20] Gulati S, Bansal A, Pal A, et al. Estimating PM2. 5 utilizing multiple linear regression and ANN techniques. Scientific Reports, 2023, 13(1): 22578.

[21] Gill R, Greenland S. Branding and the Olympics. Asia Pacific Public Relations Journal, 2024, 26.

[22] Kumar R. Building a healthy nation: A white paper on Olympic sports and the Indian education system. Journal of Family Medicine and Primary Care, 2024, 13(8): 2805-2818.

[23] Mu'ammal I, Muzakki A, Fakhri E A, et al. The competence of a coach in sports: How does it correlate with athlete motivation?. Journal Sport Area, 2022, 7(3): 396-404.

Robin M. Hogarth, Departament d'Economia i Empresa, Universitat Pompeu Fabra Barcelona Graduate School of Economics

Wei Miao, Departament d'Economia i Empresa, Universitat Pompeu Fabra Barcelona Graduate School of Economics

David T., Departament d'Economia i Empresa, Universitat Pompeu Fabra Barcelona Graduate School of Economics

A. W. Henry, Departament d'Economia i Empresa, Universitat Pompeu Fabra Barcelona Graduate School of Economics

Manuscript Published; 09 May 2025.